

# SIMPLE NONLINEAR STATIC ANALYSIS PROCEDURE FOR PROGRESSIVE COLLAPSE EVALUATION



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**Abstract:** There is a concern about progressive collapse of buildings. A simple structural design criterion including definitions for key or important members in a structure is proposed and a single-degree-freedom model is created first to illustrate the analysis procedure of progressive collapse. Then, a nonlinear static analysis procedure for existing buildings is presented. Evaluation of a six-story concrete structure is carried out based on this procedure and the result of this simplified approach is compared with the calculation from a nonlinear dynamic procedure.

**Key Words:** Progressive Collapse, Criterion, Nonlinear, Static

## INTRODUCTION

After the Murrah Building collapse in Oklahoma City, there is an interest in progressive collapse potential evaluation. The General Services Administration issued “Progressive Collapse Analysis and Design Guidelines – for New Federal Office Buildings and Major Modernization Projects” (GSA, 2003) as a guideline for engineers involved in the evaluations. The guidelines and additional information including standards, references, test data, computer programs and reference projects can be found at the GSA website <http://www.oca.gsa.gov/>. More information can also be found in other codes including the New York City Code Chapter 18 of Rules and Regulations, The British code and the ACI code. One requirement common to various codes is to verify the presence of an alternate load path in case a column is removed and some prescriptive requirements are not met. Another is to strengthen the key or important members defined as those whose failure can create extensive damage to the structure.

The goal of a designer is to produce designs that are reliable and to avoid creating structures that can have major collapses due to damage in small areas or failure of single elements. To realize these designs engineers should have an overall concept for the structural design, should determine what members are important or key to their design, and have the tools to assess the extent of structural damage if a structural member fails.

The designer should be aware that in an optimally designed structure every square foot has the same reliability for every accidental loading case considered. If this is not the case, one part of the structure is stronger than another and the designer should reinforce the area more prone to fail with material from the area less prone to fail. In practice is most probably impossible to obtain the same reliability everywhere hence the goal should be to obtain a minimum allowable reliability for every square foot of structure.

The designer can identify the key or important members in a structure, as the ones that have bigger influence area and/or carry more loads and/or have higher strain energy and/or those whose failure can create extensive collapse. The bigger the load a member carries or the bigger the influence area the more load the structure has to redistribute or the bigger is the extent of the damage in case the member fails. The strain energy under dead loads is a measure of the work of the member; the more work the member does the more significant is the member. As the strain energy is the combination of the load and the section properties this is a more complex indicator than the total load or the influence area that are only function of one parameter. Other important indicator is the reserve capacity of the member defined as the ratio of the energy the member can absorb until it fails to the strain energy it has under dead loads. Another indicator is the ratio of the strain energy of the member to the volume of the member or strain energy density that gives an idea of what members are more stressed out. In an ideal structure this density is uniform throughout the structure. The strain energy density is a more exact indicator than the stress level because it takes into consideration the whole volume of the member not just a section. Finally the failure of secondary members that brace or stabilize the primary members can produce significant damage to the structure. These members have a small influence area, carry small loads and have small strain energy. These members cannot be easily associated to a numerical parameter. The designer will identify these members also as key or important to the redundancy of the structure.

The rest of this paper explains to the designer some fundamental concepts to compute the structural behavior under a column removal scenario.

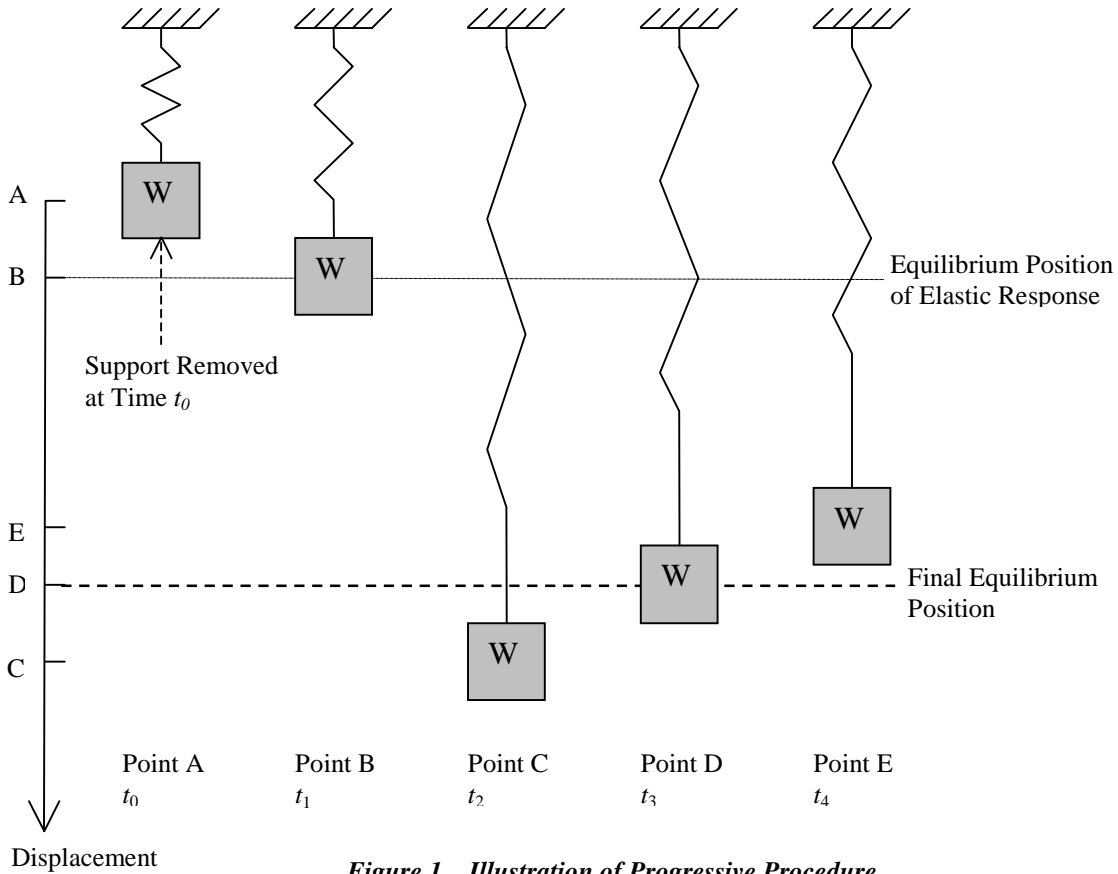
This paper is a further exploration of the energy balance method used by Graham H. Powell in “Collapse Analysis Made Easy (More or Less)” (Graham H. Powell, 2003). The method is the technical basis for the RAM Perform-Collapse computer program

In this paper, a single-degree-of-freedom nonlinear system, consisting of a nonlinear spring and a concentrate mass, is created first to illustrate the procedure of progressive collapse. Section I is the detailed description. Section II presents a nonlinear static analysis procedure for existing buildings. The basic concept of the procedure is energy balance, i.e., the structure must absorb the potential energy generated due to the removal of one column. Section III is an example to illustrate the above procedure. Evaluation of a six-story concrete structure is carried out and the result is compared with a nonlinear dynamic procedure

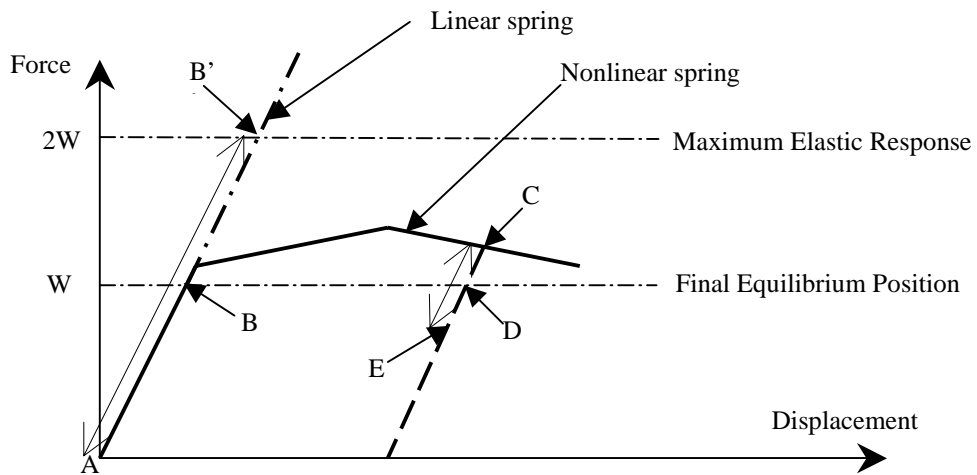
## I. ILLUSTRATION OF PROGRESSIVE COLLAPSE

The progressive collapse procedure is similar to a single-degree-freedom system as shown in Figure 1. Figure 2 is the property of the nonlinear spring. Point A, B, C, D and E in Figure 1 and Figure 2 denote same state. Table 1 is the list of system variables.

Energy dissipated in the structure due to damping is minimum compared with the energy absorbed due to plastic deformation. Thus, damping is not considered in the following description of the progressive collapse procedure.



**Figure 1. Illustration of Progressive Procedure**



**Figure 2. Force vs. Displacement Diagram of Spring**

**Table 1 System Variable**

Point	Force	Potential Energy	Kinetic Energy	Energy absorbed by Spring
A	Down	$-W*A^a$	0	0
B	Zero	$-W*B^a$	+	+
C	Up	$-W*C^a$	0	$W*C^a$
D	Zero	$-W*D^a$	+	+
E	Down	$-W*E^a$	0	$W*E^a$

<sup>a</sup>: A, B, C, D, and E denote the displacement coordinate at those points.

- At point A, when the column is removed, the system has the maximum potential energy. Since the force in the spring is zero at this time, the system is falling down due to the weight of the system, W.
- From point A to B, the downward velocity increases and reaches its maximum at point B. After point B, the downward velocity decreases because the force in the spring is greater than the weight of the system, W. If the yield capacity is greater than 2W, the response of the system is linear static as the straight line AB' shown in Figure 2.
- At point C, the falling system has zero velocity and all the potential energy is absorbed by the spring. Point C can be obtained by above energy balance condition. After point C, the system starts rebound because force in the spring is greater than the weight of the system, W.
- At point D, the system has maximum upward velocity. From point D to point E, the upward velocity decreases and becomes zero at point E. If the unloading curve of the spring is straight, it can be seen that distance CD equal to DE.
- Point D will be the final state.

Several conclusions can be reached:

- For the system not to fail, the strength of the spring at point C must be greater than the weight of the system.
- If the weight of the system is greater than the maximum strength capacity of the spring, the system will fail.
- If the weight of the system is smaller than half of the yield strength of the spring, the system has only elastic response and will not collapse.
- The magnitude of the vibration between point C and point E is generally small compared with the elastic response and generally there is no load reversal. Hence the system will not fail as it oscillates around point D.

## II. NONLINEAR STATIC ANALYSIS PROCEDURE

Following is a description of the proposed nonlinear static analysis procedure:

1. Put a load proportional to the reaction of the removed column and increase it gradually to get the pushover curve of the structure.
2. If the reaction is less than half of the yield strength of the pushover curve, the structure has low potential for progressive collapse.



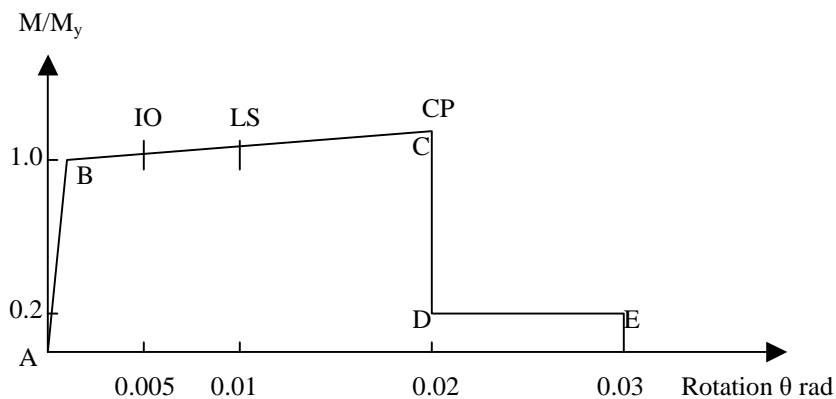
**Table 2 Six-Story Building Beam Reinforcement and Capacity**

Floor	Beam Size		Flange Width (in)	Flange Add. Top Rebar (in <sup>2</sup> )	Top Bar (in <sup>2</sup> )	Bottom Bar (in <sup>2</sup> )	Stirrup		Shear Cap kip	Positive Mom. (k-f)	Negative Mom. (k-f)
	Depth (in)	Width (in)					(in <sup>2</sup> )	Space			
RF	39	14	72	1.76	1.2	1.58	0.22	12	118	359	642
6	29.5	14	32	1.24	1.2	1.58	0.22	12	87	262	389
5	29.5	14	32	1.24	1.2	1.58	0.22	12	87	262	389
4	29.5	14	32	1.24	1.2	1.58	0.22	12	87	262	389
3	29.5	14	32	1.24	1.2	1.58	0.22	12	87	262	389
2	49.5	14	52	1.86	1.2	2.54	0.22	12	152	739	863

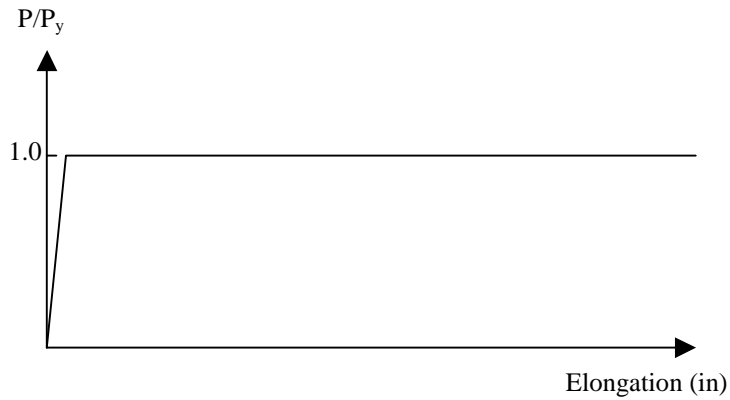
**Table 3 Existing Column Size and Reinforcement**

Floor	Six-Story Building			
	Middle Col		Corner Col	
	Size (in)	Rebar	Size	Rebar
6	40X16	6#7	L shape with each leg 40X16	12#7
5		6#7		12#7
4		6#8		12#7
3		6#8		12#7
2		6#9		12#7
1		6#10		12#7

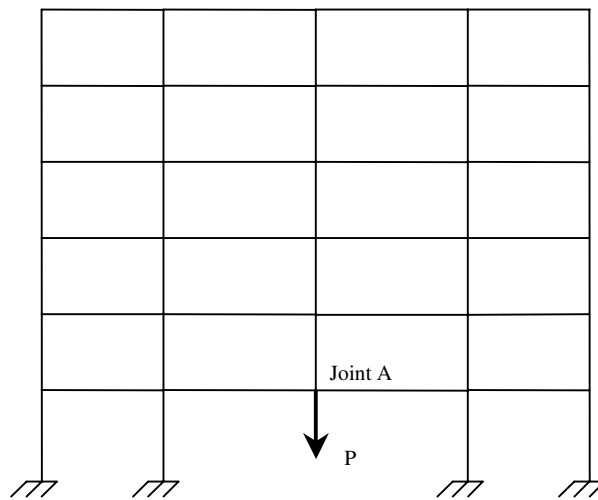
Plastic moment hinges and axial hinges are assigned to beam ends. Moment hinge properties are taken from FEMA 356 (FEMA, 2000) as shown in Figure 4. Figure 5 is the axial hinge property diagram, assuming infinite deformation capacity.  $P_y$  for the beams is calculated taking only into consideration the rebar located in the area that is in compression due to flexure. We assume that the rebar located in the area that is in tension due to flexure has yielded. Columns are assumed to remain elastic due to their size. The hinge modeling does not account for interaction between the axial force and the bending moment. Large displacement analysis is used to engage cable action. Horizontal supports are provided to simulate restraint actions of the other bents. The actual solution is bracketed between having horizontal supports that enhance cable action and decrease column bending and having no horizontal supports that neglects any restraint provided by the floor slab.



**Figure 4. Moment Hinge Properties**

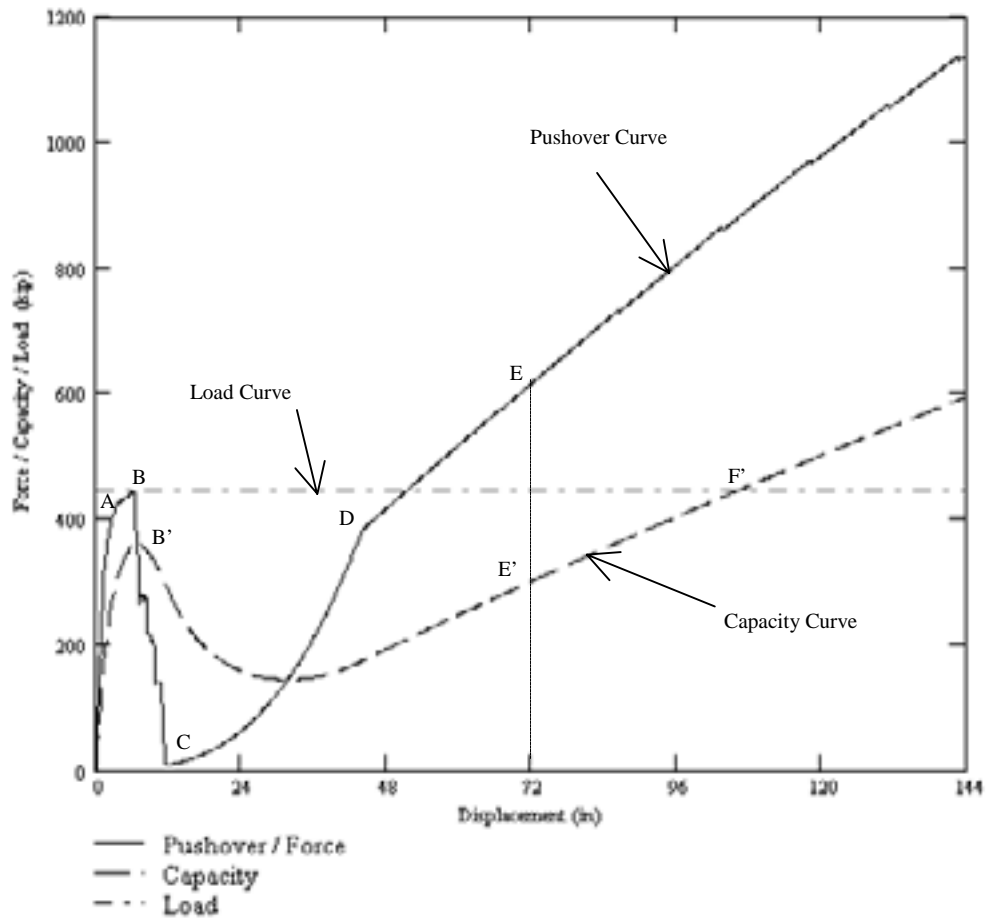


**Figure 5. Axial Hinge Properties**



**Figure 6. Loading for Pushover Analysis Procedure**

Figure 6 shows the loading condition to get the pushover curve. For simplicity, the structure is set up with no gravity load and a missing column. A more accurate solution is to include all the gravity loads present at the time of the column removal. The load  $P$  is equal to the reaction of the column removed, 440 kips in this case. For this example, we apply a maximum displacement of 144". The displacement control analysis computes at each displacement step the amount of load required to create the displacement.



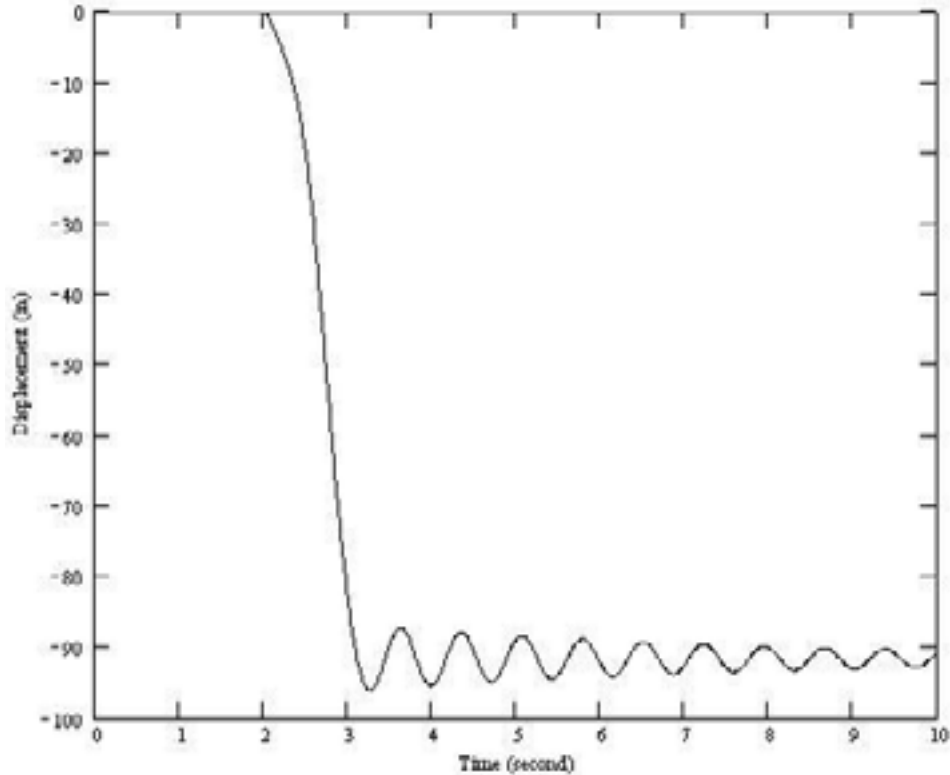
**Figure 7. Pushover Curve, Capacity Curve, and Load Curve**

Figure 7 is the pushover curve. Point A, B, C, D, and E on the pushover curve indicates different stages of structure behavior. Before point A, the structure behaves elastically with point A corresponding to the yielding of the structure. After yielding, the beams strength hardened from point A to B. At point B, the hinges fail and there is an abrupt drop. Curve CD indicates that the structure begins to pick up load due to cable action. At point D, reinforcement bars yield due to tension and the slope of the pushover curve becomes smaller. Since the model assumes the rebar has infinite deformation capacity, the structure can continue to sustain load without failure.

The area below the pushover curve is the energy that the structure can absorb. If we divide the energy below the pushover curve by the corresponding displacement, we can get the capacity curve of the structure. For example, point E' on the capacity curve is obtained by dividing area below OABCDE by the displacement at E, 72" in this case. The pushover curve and capacity curve are characteristics of the structure under given load condition.

The load curve is straight in this case, which is equal to the reaction of the removed column, 440 kips in this case. From Figure 7, it can be seen that the capacity curve is lower than the load curve before point F', which means that the structure can not absorb the potential energy before reaching the displacement corresponding to point F'. It is obvious that the structure will collapse if it deflects as much as point F', even if the energy can be balanced at point F. Thus, the conclusion is that the 2-D frame has a high potential for progressive collapse.

Figure 7 also shows that the capacity of the structure is about 80% of the required capacity near point B'.



**Figure 8. Vertical Displacements vs. Time Diagram**

A nonlinear dynamic analysis is also carried out to verify the result of the nonlinear static procedure above. 3% of critical damping is introduced and force equal to the reaction of the column is put on the structure. Figure 8 is the joint vertical displacement response, where the column is removed.

The structure will not collapse because the axial hinge has infinite deformation capacity. The structure will reach its equilibrium at about 93 inches, which is close to 100 inches from the previous nonlinear static analysis. If the load on the structure is reduced to about 80%, the moment hinges will not fail, which is also very close to the nonlinear static analysis.

#### IV. CONCLUSION

The following conclusions can be reached:

- A simple structural design criterion including definitions for key or important members in a structure is proposed.
- A simple quantitative nonlinear static procedure is proposed for analyzing the progressive collapse potential caused by the removal of a column.
- The proposed nonlinear static procedure gives reasonable results for the example shown. The procedure also gives a quantitative measurement of the potential for progressive collapse.

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